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# A METHOD FOR CALCULATION OF HEAT TRANSFER COEFFICIENTS FOR OPTIMIZATION OF THERMOELECTRIC COOLERS

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*A method has been developed for calculation of heat transfer coefficients for various kinds of surfaces in a multistage thermoelectric cooler exchanging heat with gaseous media and bodies. Heat inleakages and heat transfer coefficients have been estimated for coolers of the Aryk type in air, xenon, and argon. A computer program has been written and computations have been carried out for determination of heat transfer coefficients and heat inleakages to a working site, lateral surfaces of thermoelements, and to inner and free areas in heat leveling plates in coolers of the Aryk and Manchak types under different conditions and in different gaseous media.*

In solid-state coolers with photosensitive elements (PSE) in vacuum or gaseous media at low temperature, heat inleakages through various structural elements become important and can bring about substantial changes in the parameters and cause instability of the system. Direct measurement of heat inleakages is difficult. Therefore, it is very important to calculate and estimate the maximum value of these heat inleakages and to investigate the possibility of decreasing them to maintain stable operation of the device. For this purpose, first of all, it is necessary to determine the heat transfer coefficients under different conditions.

In the present work a method is suggested for calculation of heat transfer coefficients for two models of coolers which are used in practice (Fig. 1). The first model is a four-stage thermoelectric cooler (TEC). On the working site of the TEC a ceramic board is mounted for soldered connections of the leads of the thermosensitive element and a temperature-sensitive resistor for temperature control. The cooler is placed in a vacuum tank. In the second model a heat reflecting screen is mounted in the third stage of the thermal cooler. A screen shaped as a truncated pyramid protects the two upper stages from radiation of the heat generating base and inner walls of the case. The convective heat flux to the TEC is decreased by both evacuation and filling the working space of the cooler with an insert gas, for example, xenon or argon.

Heat is supplied to the working site via the leads of the soldered connections of the PSE and by convection, conduction, and radiation through the gaseous medium from the vacuum tank and through the sapphire window. The working site ($7.3 \times 3$ mm) is located at distance $\delta_0 = 1.6$ mm from the top part of the vacuum tank so that heat transfer occurs in a confined space, which, with the present dimensions, can be considered a horizontal slot filled with a gaseous medium. When the heated surface is located at the top, convection is absent in the horizontal slots, and heat is transferred by conduction through a gas interlayer with thickness $\delta_0$. In this case the heat transfer coefficient $\alpha_{\text{med}} = \lambda_{\text{med}}/\delta_0$ and the heat transferred by contact

$$Q_{\text{med}} = \alpha_{\text{med}} S_s (T_{\text{med}} - T_0) ,$$

where $\lambda_{\text{med}}$ is the thermal conductivity of the medium; $S_s$ is the surface area of the working site; $T_{\text{med}}$ is the temperature of the medium; $T_0$ is the temperature of the working site. The thermal conductivity is determined at the average temperature of the medium $\overline{T} = 1/2(T_{\text{med}} + T_0)$.

Radiation heat transfer between two bodies located arbitrarily in the space is described by the formula [1]:

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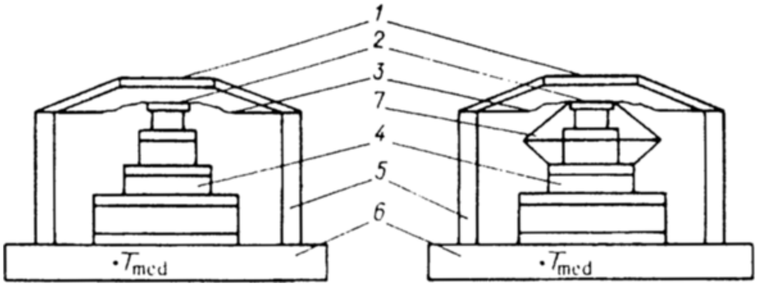


Fig. 1. Models of thermoelectric coolers: a) model No. 1; b) model No. 2, [1) lid of vacuum cap with a saphire window; 2) working site of TEC; 3) leads from plate to PSE; 4) thermoelectric cooler; 5) vacuum cap; 6) base; 7) screen.

TABLE 1. Estimated Heat Transfer Coefficients of Structural Elements of Aryk Cooler and Medium for a Preset Temperature Distribution in Stages ($T_0$ = 190 K, $T_1$ = 209 K, $T_2$ = 233 K, $T_3$ = 261 K, $T_{med}$ = 298 K)

| Medium | Working site $\alpha_0 \cdot 10^3$, W/(cm$^2\cdot$K) | $\alpha_{lat} \cdot 10^3$, W/(cm$^2\cdot$K) | $\alpha_{in} \cdot 10^3$, W/(cm$^2\cdot$K) | | $\alpha_{fr} \cdot 10^3$ | $\alpha_{e.f} \cdot 10^3$ |
|---|---|---|---|---|---|---|
| | | | upward inleakages | downward inleakages | W/(cm$^2\cdot$K) | |
| Air | 1.651 ($\varepsilon_1$ = 0.6)<br>1.767 ($\varepsilon_1$ = 0.9) | 2.6 | 8.0 | 4.2 | 2.6–1.84 | 4.5 |
| Argon | 1.172 ($\varepsilon_1$ = 0.6)<br>1.285 ($\varepsilon_1$ = 0.9) | 2.0 | 5.8 | 3.05 | 1.4 | 3.35 |
| Xenon | 0.512 ($\varepsilon_1$ = 0.6)<br>0.619 ($\varepsilon_1$ = 0.9) | 1.09 | 2.75 | 1.56 | 0.92 | 1.7 |

$$Q^r_{12} = \sigma\varepsilon_{red}(T_1^4 - T_2^4)H_{12},$$

where $\varepsilon_{red}$ is the reduced emissivity of the system:

$$\varepsilon_{red} = \left[\frac{1}{\varepsilon_1} + \frac{F_1}{F_2}\left(\frac{1}{\varepsilon_2} - 1\right)\right]^{-1};$$

$\varepsilon_1$ and $\varepsilon_2$ are the emissivities of surfaces 1 and 2; $\sigma$ is the Stefan–Boltzmann constant; $H_{12}$ is the mutual radiation surface of the bodies; $T_1$ and $T_2$ are the temperatures of the bodies; $F_1$ and $F_2$ are the surfaces of the bodies. The radiation heat transfer $\alpha_r$ is defined by the formula

$$\alpha_r = \sigma\varepsilon_{red}\frac{T_1^4 - T_2^4}{T_1 - T_2},$$

and the total heat transfer coefficient in the medium is $\alpha_0 = \alpha_{med} + \alpha_r$.

The heat inleakages to the working site of the cooler for the various media and estimated heat transfer coefficients are given in Table 1 (with preset temperature distributions in the TEC stages).

Heat inkeakages to the lateral surface of the thermoelements can be considered as a flow (natural convection) of gas with temperature $T^{(i)}_{med} = (1 - k_i)T_{med} + k_i\overline{T}$ over a vertical panel with temperature $\overline{T} = 1/2(T_i + T_{i-1})$ ($i$ is the stage number) (Fig. 2a). Here $k_i = s_0/(s_0 + s_1)$ is the compactness factor; $s_1$ is the area of the heat-leveling plate per branch. In this case, in calculation of the similarity numbers, the height of the plate $l$ is taken as the characteristic dimension and the average temperature of the boundary layer $T_m$ =

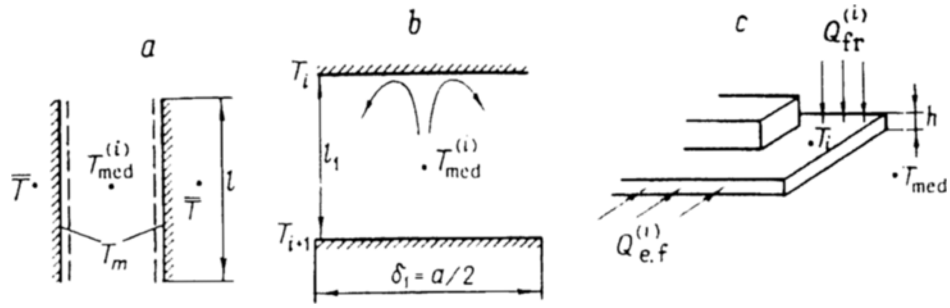


Fig. 2. Diagram of heat transfer of structural elements of TEC to the environment: a) from lateral surfaces of thermocouples; b) from inner free areas of heat-leveling plates; c) from free end faces of heat-leveling plates.

$1/2(T_{med}^{(i)} + \bar{T})$ is taken as the characteristic temperature. In this case the convective heat transfer coefficient is $\alpha_{med}^{lat} = Nu_m \lambda_m / l$, where $l$ is the current length of the branch, $\lambda_m$ is the thermal conductivity of the medium at the temperature $T_m$ and $Nu_m$ is the Nusselt number. The radiation heat transfer coefficient is $\alpha_r^{lat} = \sigma\varepsilon_{red}(T_{med}^{(i)} - \bar{T})^4/(T_{med}^{(i)} - \bar{T})$. The total heat transfer coefficient between the lateral surfaces of the thermocouples and the medium in the $i$-th stage of the TEC is $\alpha_{lat}^{(i)} = (\alpha_{med}^{lat})^{(i)} + (\alpha_r^{lat})^{(i)}$.

Heat transfer with the inner free areas of the heat-leveling plates of the cooler (Fig. 2b) can be considered as heat transfer to a cold medium from two plane horizontal panels, one of which faces down with its cold side and has circulation near it, and the other faces up with its clod surface and has no circulation near it. In the former case the contact heat transfer coefficiènts are 30% higher, and in the former, they are 30% lower [2]. The smaller side of the plates $\delta_1$ is taken as the characteristic dimension and $T_m^{(i)} = 1/2(T_{med}^{(i)} + T_i)$ is taken as the characteristic temperature.

Heat inleakages to free areas of the surface of the heat-leveling plates of the REC can be considered as heat supply from the environment to the horizontal plates (Fig. 2c). The cold surface of the plate faces up, there is no circulation of the medium, and $\alpha_s$ should be decreased by 30% in calculations. The characteristic dimension is the smaller side of the free part of the plate, and $T_m = 1/2(T_{med} + T_i)$ is taken as the characteristic temperature. As before, radiation heat transfer is taken into consideration. The two upper stages of the TEC considered are equal in size, and between them there are no heat inleakages from the medium to the plate.

The end face of the heat-leveling plates of the cooler can be considered as a vertical panel in a gas flow (Fig. 2c); its height is taken as the characteristic dimension and $T_m^{(i)} = 1/2(T_{med} + T_i)$ as the characteristic temperature.

Since the TEC, screen, and the vacuum tank are a heat transfer system, and the distributions of temperatures, including the temperature of the working site, depend considerably on the mutual effect of the components of this system, it seems useful to develop a mathematical model for calculation of the entire system on a computer, taking all its interactions into consideration, if possible.

For the present optimal design of an $N$-stage thermoelectric cooler (of the type shown in Fig. 1), the temperature distribution in the stages is found by solving a system of $(N - 1)$ equations by the heat-balance method, in which in the case of a steady-state temperature distribution the quantity of heat supplied to the $i$-th stage from the heat-releasing junctions of the $(i - 1)$-th stage is removed completely by the heat-absorbing junctions of the $i$-th stage. On the heat-absorbing junctions of the coldest stage the heat balance with the environment and the bodies is satisfied by removal of heat inleakages to the working site by conduction, convection, and radiation.

Moreover, in the calculation of TEC heat transfer from the lateral surfaces of thermocouples to the environment is included

$$Q_{lat}^{(i)} = n_i (1 - k_i)\, b \sqrt{s_0}\, l\, \alpha_{lat}^{(i)} \left( T_{med}^{(i)} - \frac{T_i + T_{i-1}}{2} \right),$$

TABLE 2. Quantity of Heat Inleakage and Heat Transfer Coefficients for Aryk Cooler Working in Air Medium (effect of control thermocouple neglected)

| Inleakage heat, heat transfer coefficients | Working site | | Stage No. | Lateral surfaces | Inner surfaces | | Free surfaces | Temperature distribution in stages, $T_i$, K |
|---|---|---|---|---|---|---|---|---|
| | $l = 72$ | $l = 52$ | | | upper sites | lower sites | | |
| $Q^{(i)}$, mW | 50.46 | 47.60 | I | 2.162 | 47.982 | 17.550 | – | 196 |
| | | | II | 1.738 | 71.722 | 24.370 | – | 216 |
| | | | III | 1.260 | 137.81 | 30.650 | 73.660 | 234.3 |
| | | | IV | 0.503 | 265.35 | −22.940 | 83.410 | 258.6 |
| $\alpha_0^{(i)} \cdot 10^3$, W/(cm$^2$·K) | 1.754 | | I | 2.580 | 8.0 | 4.34 | – | 196 |
| | | | II | 2.620 | 8.05 | 4.29 | – | 216 |
| | | | III | 2.650 | 7.97 | 4.26 | 2.590 | 234.3 |
| | | | IV | 2.604 | 8.20 | 2.85 | 1.839 | 258.6 |
| $\alpha_{med}^{(i)} \cdot 10^3$, W/(cm$^2$·K) | 1.394 | | I | 2.295 | 7.74 | 4.04 | – | 196 |
| | | | II | 2.272 | 7.74 | 3.94 | – | 216 |
| | | | III | 2.242 | 7.59 | 3.83 | 2.160 | 234.3 |
| | | | IV | 2.085 | 7.74 | 2.27 | 1.344 | 258.6 |
| $\alpha_r^{(i)} \cdot 10^3$, W/(cm$^2$·K) | 0.360 | | I | 0.286 | 0.269 | 0.304 | – | 196 |
| | | | II | 0.338 | 0.321 | 0.356 | – | 216 |
| | | | III | 0.403 | 0.378 | 0.431 | 0.438 | 234.3 |
| | | | IV | 0.520 | 0.468 | 0.577 | 0.494 | 258.6 |

TABLE 3. Quantities of Heat Inleakage and Heat Transfer Coefficients for Manchak Cooler Working in Air Medium (effect of control thermocouple neglected)

| Inleakage heat, heat transfer coefficients | Working site $l = 52$ | Stage No. | Lateral surfaces | Inner surfaces | | Free surfaces | Temperature distribution in stages | |
|---|---|---|---|---|---|---|---|---|
| | | | | upper sites | lower sites | | $\alpha^{(i)} = \alpha^{(i)}(T_i)$ | $\alpha^{(i)} = \mathrm{const} = H$ |
| $Q^{(i)}$, mW | 47.75 | I | 1.694 | 40.130 | 14.370 | – | 195.13 | 191.42 |
| | | II | 1.349 | 59.300 | 19.940 | – | 216.40 | 215.92 |
| | | III | 0.971 | 113.85 | 24.640 | 55.930 | 234.80 | 235.26 |
| | | IV | 0.382 | 215.60 | −19.080 | 62.680 | 259.30 | 260.70 |
| $\alpha_0^{(i)} \cdot 10^3$, W/(cm$^2$·K) | 1.753 | I | 2.971 | 8.681 | 4.694 | – | 195.13 | 191.42 |
| | | II | 2.996 | 8.737 | 4.631 | – | 216.40 | 215.92 |
| | | III | 3.031 | 8.639 | 4.587 | 2.593 | 234.80 | 235.26 |
| | | IV | 2.960 | 8.863 | 3.174 | 1.838 | 259.30 | 260.70 |
| $\alpha_{med}^{(i)} \cdot 10^3$, W/(cm$^2$·K) | 1.393 | I | 2.685 | 8.412 | 4.390 | – | 195.13 | 191.42 |
| | | II | 2.656 | 8.415 | 4.273 | – | 216.40 | 215.92 |
| | | III | 2.625 | 8.259 | 4.153 | 2.154 | 234.80 | 235.26 |
| | | IV | 2.434 | 8.393 | 2.596 | 2.342 | 259.30 | 260.70 |
| $\alpha_r^{(i)} \cdot 10^3$, W/(cm$^2$·K) | 0.359 | I | 0.286 | 0.269 | 0.304 | – | 195.13 | 191.42 |
| | | II | 0.340 | 0.323 | 0.358 | – | 216.40 | 215.92 |
| | | III | 0.406 | 0.380 | 0.434 | 0.439 | 234.80 | 235.26 |
| | | IV | 0.525 | 0.470 | 0.577 | 0.497 | 259.30 | 260.70 |

TABLE 4. Quantities of Heat Inleakage and Heat Transfer Coefficients of Aryk CoolVler Working in Deep Vacuum with a Mirror Working Site (effect of control thermocouple neglected)

| Inleakage heat, heat transfer coefficients | Working site | | Stage No. | Lateral surfaces | Inner surfaces | | Free surfaces | Temperature distribution in stages | |
|---|---|---|---|---|---|---|---|---|---|
| | $l=52$ | $l=72$ | | | upper sites | lower sites | | $l=52$ | $l=72$ |
| $Q^{(i)}$, mW | 8.63 | 11.93 | I | 0.243 | 1.626 | 1.269 | – | 178 | 178.4 |
| | | | II | 0.237 | 3.030 | 2.131 | – | 201.9 | 202.06 |
| | | | III | 0.206 | 7.168 | 3.234 | 13.60 | 224.94 | 225.00 |
| | | | IV | 0.108 | 16.360 | −4.902 | 24.07 | 255.06 | 255.08 |
| $\alpha^{(i)} = \alpha^{(i)}_{\mathrm{med}} \cdot 10^3$, $\mathrm{W/(cm^2 \cdot K)}$ | – | | I | 0.249 | 0.231 | 0.268 | – | 178 | 178.4 |
| | | | II | 0.308 | 0.288 | 0.329 | – | 201.9 | 202.06 |
| | | | III | 0.387 | 0.355 | 0.420 | 0.419 | 224.94 | 225.00 |
| | | | IV | 0.516 | 0.461 | 0.576 | 0.489 | 255.06 | 255.08 |

TABLE 5. Quantities of Heat Inleakage and Heat Transfer Coefficients for Aryk Cooler Working in Air without Radiation (effect of control thermocouple neglected)

| Inleakage heat, heat transfer coefficients | Working site | | Stage No. | Lateral surfaces | Inner surfaces | | Free surfaces | Temperature distribution in stages | |
|---|---|---|---|---|---|---|---|---|---|
| | $l=52$ | $l=72$ | | | upper sites | lower sites | | $l=52$ | $l=72$ |
| $Q^{(i)}$, mW | 17.33 | 20.58 | I | 1.943 | 46.94 | 16.38 | – | 194.24 | 194.52 |
| | | | II | 1.527 | 69.32 | 22.39 | – | 215.24 | 215.38 |
| | | | III | 1.071 | 131.59 | 27.08 | 61.64 | 233.90 | 233.90 |
| | | | IV | 0.402 | 249.33 | −18.51 | 60.98 | 258.60 | 258.60 |
| $\alpha^{(i)} = \alpha^{(i)}_{\mathrm{med}} \cdot 10^3$, $\mathrm{W/(cm^2 \cdot K)}$ | 1.39 | | I | 2.296 | 7.736 | 4.041 | – | 194.24 | 194.52 |
| | | | II | 2.277 | 7.736 | 4.041 | – | 215.24 | 215.38 |
| | | | III | 2.241 | 7.584 | 3.834 | 2.158 | 233.90 | 233.90 |
| | | | IV | 2.087 | 7.744 | 2.281 | 1.346 | 258.60 | 258.60 |

TABLE 6. Quantities of Heat Inleakage and Heat Transfer Coefficients for Aryk Cooler Working in Deep Vacuum (effect of control thermocouple neglected)

| Inleakage heat, heat transfer coefficients | Working site | | Stage No. | Lateral surfaces | Inner surfaces | | Free surfaces | Temperature distribution, $T_i$, K |
|---|---|---|---|---|---|---|---|---|
| | $l=52$ | $l=72$ | | | upper sites | lower sites | | |
| $Q^{(i)}$, mW | 17.33 | 20.58 | I | 0.244 | 1.628 | 1.273 | – | 179.0 |
| | | | II | 0.237 | 3.030 | 2.136 | – | 202.5 |
| | | | III | 0.206 | 7.180 | 3.240 | 13.60 | 225.2 |
| | | | IV | 0.107 | 16.340 | −4.900 | 24.04 | 255.2 |
| $\alpha^{(i)}_0 = \alpha^{(i)}_r \cdot 10^3$, $\mathrm{W/(cm^2 \cdot K)}$ | 0.332 | | I | 0.251 | 0.234 | 0.269 | – | 179.0 |
| | | | II | 0.309 | 0.289 | 0.330 | – | 202.5 |
| | | | III | 0.387 | 0.356 | 0.421 | 0.42 | 225.2 |
| | | | IV | 0.516 | 0.462 | 0.576 | 0.49 | 255.2 |

where $n_i$ is the number of thermocouple branches in the $i$-th stage; $b = P/\sqrt{s_0}$ is the shape factor; $P$ is the perimeter of the branch. It is possible to show that almost half of $Q_{lat}^{(i)}$ is supplied to the cold junctions and half to the hot junctions.

The heat inleakage to free surfaces of the heat-leveling plate between the $i$-th and $(i + 1)$-th stages is determined by the expression:

$$Q_{fr}^{(i)} = \frac{s_0}{k_i} (n_{i+1} - n_i) \alpha_{fr}^{(i)} (T_{med} - T_i) .$$

To the inner surfaces between the branches of the thermocouples the heat

$$Q_{in}^{(i)} = \alpha_{in}^{(i)} s_1 \ [n_i (T_{med}^{(i)} - T_i) + n_{i+1} (T_{med}^{(i+1)} - T_i)] ,$$

is supplied and to the surfaces of the end faces of the heat-leveling plates the heat

$$Q_{e\,f}^{(i)} = \alpha_{e\,f}^{(i)} S_{e\,f}^{(i)} (T_{med} - T_i) ,$$

is supplied, where $S_{e\,f}^{(i)}$ is the area of the end face of the heat-leveling surface in the $i$-th stage.

Thus, heat transfer from the branches and heat-leveling plates of the TEC to the environment and bodies is connected with the temperature distribution in the stages, and the distribution depends, in turn, on the conditions of heat exchange with the environment, since an increase in the heat load on the TEC increases sharply the temperature of its cooling junctions. Therefore, the heat transfer coefficients change from stage to stage. In view of this, it is necessary to include data on temperature-dependent quantities in the program for cooler design.

For the second model the inleakage heat $Q_{sc}$ from the surface of the screen $S_{sc}$ to the third stage of the cooler should be taken into consideration. Since the temperature of the screen $T_{sc}$ is close to the temperature of the third stage, the total heat transferred to the screen from the medium by convection and radiation

$$Q_{sc} = \alpha_{med} S_{sc} (T_{med} - T_{sc}) + \sigma \varepsilon_{red} (T_{med}^4 - T_{sc}^4)$$

It should be removed additionally by the lower stages of the cooler.

As an example of the feasibility of the present method, two real four-stage coolers corresponding to the first model were calculated. The heat transfer coefficients and heat inleakages to the structural elements of four-stage coolers of types Aryk ($l$ = 1.8 mm; $s_0$ = 0.6 × 0.8 mm) and Manchak ($l$ = 1.4 mm; $s_0$ = 0.7 × 0.7 mm) are given in Tables 2 and 3. The numbers of branches in the stages of the TEC are $n_1$ = 12, $n_2$ = 22, $n_3$ = 52, $n_4$ = 128. Results of these calculations agree well with the estimates from Table 1. In Table 3, for comparison, the temperature distribution in the stages of the cooler are given with the heat transfer coefficient $H$ as a function of temperature and the type of surface taken into consideration. The results of these calculations differ slightly, which can be explained by the fact that constancy of the heat transfer coefficients $H$ is a rougher approximation of real conditions.

We will also consider the limiting case of deep vacuum with a mirror surface of the working site and the case of convection of the medium (Tables 4-6). From a comparison of the calculated results with experimental data it is possible to estimate the heat transfer coefficients from the quantity of heat inleakage and to compare them with the calculated coefficients. It should be also taken into consideration that in computations on a computer, the emissivity factor of the surfaces of the cooler was assumed to be equal to unity. With appropriate processing of the results it is also possible to estimate the real emissivity factor of the surfaces studied.

## NOTATION

$\alpha$, heat transfer coefficient, W/(cm$^2$·K); $\lambda$, thermal conductivity, W/(cm·K); $s_0$, cross-sectional area of branches of thermocouples, mm$^2$; $l$, current length of a branch of the thermocouple, mm; $s_1$, free surface area of the heat-leveling plate around branches of thermocouples, $mm^2$; $Q$, quantity of heat, W; $T$, temperature, K; $\sigma$, Stefan–Boltzmann constant, $W/K^4$; $b$, shape factor; $k$, compactness factor; $\epsilon$, emissivity factor; Nu, Nusselt number; $n$, the number of branches in the TEC; $i$, the stage number; $N$, the number of stages in the TEC; $l_1$, distance between free areas of heat-leveling plates, mm; $l$, the number of soldered connection leads of the PSE. Subscripts and superscripts: in, inner; fr, free; e.f, refers to the end face; lat, lateral.

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